Math 409 Midterm 1 practice

Name: ______________________________

This exam has 3 questions, for a total of 100 points.

Please answer each question in the space provided. No aids are permitted.

Question	Points	Score
1	40	
2	30	
3	30	
Total:	100	

Question 1. (40 pts)

In each of the following eight cases, indicate whether the given statement is true or false. No justification is necessary.

(a) Let E be a set and suppose that there exists a surjective function $f \colon \mathbb{R} \to E$. Then E is uncountable.

(b) If E is a subset of $\mathbb{R}$ which has a supremum, then the set $-E = \{-x \colon x \in E\}$ has an infimum.

(c) Let $a \in \mathbb{R}$. Then $|a| < \varepsilon$ for all $\varepsilon > 0$ if and only if $a = 0$.

(d) If $\{E_x\}_{x\in\mathbb{R}}$ is a collection of finite sets indexed by the real numbers, then $\bigcup_{x\in\mathbb{R}} E_x$ is at most countable.

(e) Every subset of $\mathbb{R}$ has at most two suprema.

(f) Let $f\colon \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$. Then $f^{-1}([0,1]) = [-1,1]$.

(g) Let $A_1, A_2, A_3, \cdots$ be nonempty finite subsets of $\mathbb{N}$ such that $A_n \cap A_m = \emptyset$ for all distinct $n, m \in \mathbb{N}$. Define the function $f\colon \mathbb{N} \to \mathbb{N}$ by declaring $f(n)$ to be the least element of A_n. Then f is injective.

(h) Let $A_1, A_2, A_3, \cdots$ be nonempty bounded subsets of $\mathbb{R}$ such that $A_n \cap A_m = \emptyset$ for all distinct $n, m \in \mathbb{N}$. Define the function $f\colon \mathbb{N} \to \mathbb{R}$ by $f(n) = \sup A_n$. Then f is injective.

Question 2. (30 pts)

(a) State the well-ordering principle.

(b) Prove that $2^{n-1} \leq n!$ for all $n \in \mathbb{N}$.

Question 3. (30 pts)

(a) State the completeness axiom for $\mathbb{R}$.

(b) Let A be a nonempty bounded subset of $\mathbb{R}$, and consider the set $B = \{x^2 : x \in A\}$. Prove that $\sup B$ exists.

(c) Give an example to show that the equality $\sup B = (\sup A)^2$ may fail in part (b).